

a. a_k

n

$j_k; |k| < 3$

0 otherwise

$$x(t) = -2 \sin(\pi t/2) - 4 \sin(\pi t)$$

for $0 \leq t < 4$.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k \pi t / 2}$$

$k=-\infty$

$a_k e$

j

2π

T

$$k t = -2 j e^{-j}$$

2π

4

$$2t - j e^{-j}$$

2π

4

$t + je j$

2π

4

$t + 2je j$

2π

4

$2t$

$$= -2 \sin(\pi t/2) - 4 \sin(\pi t)$$

b. $b_k =$

$\begin{cases} 1; & k \text{ odd} \\ 0; & k \text{ even} \end{cases}$

1; k odd

0; k even

$$x(t) = 2\delta(t) - 2\delta(t - 2) \text{ for } 0 \leq t < 4.$$

$$x(t) = X^\infty$$

$$k=-\infty$$

$$b_k e$$

$$j$$

$$2\pi$$

$$T$$

$$k t =$$

$$X_\infty$$

$$k = -\infty$$

$$k \text{ odd}$$

$$e$$

$$j\pi k t / 2$$

Unfortunately, this sum is not easy to close. However, it is closely related to the

synthesis formula for an impulse train,

$$x\delta(t) = X_\infty$$

$$k=-\infty$$

$$\delta(t - kT) = X_\infty$$

$$k=-\infty$$

ake

$$j\pi kt/2 =$$

$$X^\infty$$

$$k=-\infty$$

$$1$$

$$T$$

$$e$$

$$j\pi kt/2$$

.

If there were two impulses per period instead of one, then

$$x_2\delta(t) = X^\infty$$

$$k=-\infty$$

$$\delta(t - kT) + \delta(t -$$

$$T$$

$$2$$

$$-kT) = X_{\infty}$$

$$k=-\infty$$

$$1$$

$$T$$

$$e$$

$$j2\pi kt/4 +$$

$$1$$

$$T$$

$$e$$

$$j2\pi k(t-2)/4$$

$$=$$

$$X_{\infty}$$

$$k=-\infty$$

$$1$$

$$T$$

$$e$$

$$j\pi kt/2$$

2

$$1 + e$$

$$j\pi k^2$$

=

$$X_\infty$$

$$k = -\infty$$

k even

$$2$$

T

e

$$j\pi kt/2$$

It follows that $x(t) = T x\delta(t) -$

T

$$2$$

$x_2\delta(t) = 4x\delta(t) - 2x_2\delta(t)$ so that $x(t)$ is an alternating

sequence of impulses

$$x(t) = X^\infty$$

$$| = -\infty$$

$$2\delta(t - 4) - 6\delta(t - 2 - 4)$$